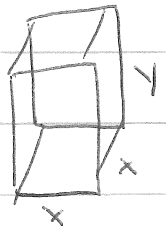
Optimization Worksheet #3

1.



$x^2y = 2100$

$y = \frac{2100}{x^2}$

$y = \frac{2100}{(13.140)^2}$

$y = 12.162$ ft.

$13.140' \times 13.140' \times 12.162'$

$C_{min} = 47x^2 + 47(4xy) + 40x^2$

$C_{min} = 87x^2 + 188xy$

$C_{min} = 87x^2 + 188x\left(\frac{2100}{x^2}\right)$

$C_{min} = 87x^2 + 394800x^{-1}$

$C' = 174x - 394800x^{-2}$

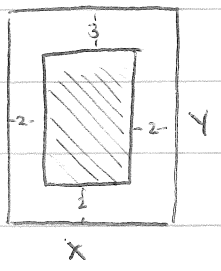
$0 = 174x - \frac{394800}{x^2}$

$\frac{394800}{x^2} = 174x$

$394800 = 174x^3$

$x = 13.140$ ft.

2.



$xy = 125$

$x = \frac{125}{y}$

$x = \frac{125}{(12.5)}$

$x = 10$ in

$A_{max} = (x-4)(y-5)$

$A_{max} = xy - 5x - 4y + 20$

$A_{max} = \left(\frac{125}{y}\right)y - 5\left(\frac{125}{y}\right) - 4y + 20$

$A_{max} = 125 - 625y^{-1} - 4y + 20$

$A_{max} = -625y^{-1} - 4y + 145$

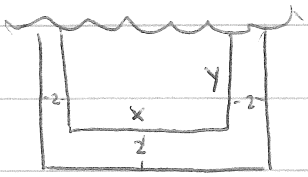
$A' = 625y^{-2} - 4$

$0 = \frac{625}{y^2} - 4$

$4 = \frac{625}{y^2}$

$y = 12.5$ in

3.



$xy = 800$

$x = \frac{800}{y}$

$x = \frac{800}{20}$

$x = 40\text{ m}$

outer rectangle:

44m x 22 m

$A_{min} = (x+4)(y+2)$

$A_{min} = xy + 2x + 4y + 8$

$A_{min} = \left(\frac{800}{y}\right)y + 2\left(\frac{800}{y}\right) + 4y + 8$

$A_{min} = 800 + 1600y^{-1} + 4y + 8$

$A_{min} = 1600y^{-1} + 4y + 808$

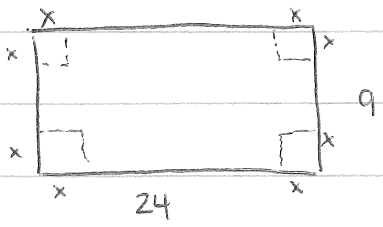
$A' = -1600y^{-2} + 4$

$0 = \frac{-1600}{y^2} + 4$

$\frac{1600}{y^2} = 4$

$y = 20\text{ m}$

4.



$l = 24 - 2x$

$w = 9 - 2x$

$h = x$

$V_{max} = lwh$

$V_{max} = (24-2x)(9-2x)x$

$V_{max} = (216 - 48x - 18x + 4x^2)x$

$V_{max} = (216 - 66x + 4x^2)x$

$V_{max} = 216x - 66x^2 + 4x^3$

$V' = 216 - 132x + 12x^2$

$0 = 12(x^2 - 11x + 18)$

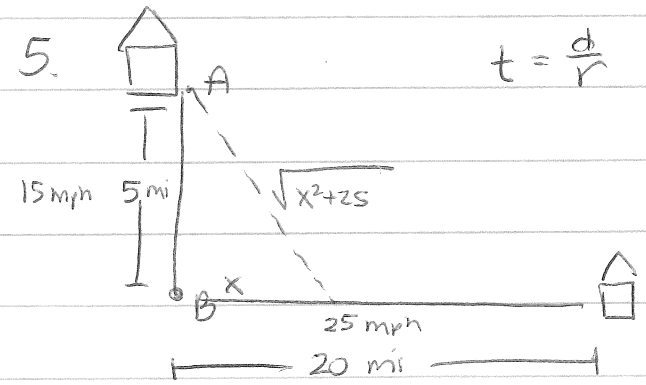
$0 = 12(x-9)(x-2)$

~~$x = 9$~~ $x = 2$

$l = 20$ in

$w = 5$ in

$h = 2$ in

5. 

$t = \frac{d}{r}$

$T_{min} = t_{lake} + t_{car}$

$T_{min} = \frac{\sqrt{x^2+25}}{15} + \frac{20-x}{25}$

$T_{min} = \frac{1}{15}(x^2+25)^{1/2} + \frac{20}{25} - \frac{1}{25}x$

$T' = \frac{1}{30}(x^2+25)^{-1/2}(2x) - \frac{1}{25}$

$0 = \frac{x}{15(x^2+25)^{1/2}} - \frac{1}{25}$

$x = 3.75$ mi

$T(0) = 1.133$ hr

$T(3.75) = 1.067$ hr

$T(20) = 1.374$ hr.

3.75 miles

6. $\frac{x^2}{4} - y^2 = 1$ to $(5,0)$

$y^2 = \frac{x^2}{4} - 1$

$y^2 = \frac{(4)^2}{4} - 1$

$y^2 = 3$

$y = \pm\sqrt{3}$

$(4, \sqrt{3})$ & $(4, -\sqrt{3})$

$d_{min} = \sqrt{(x-5)^2 + (y-0)^2}$

$d_{min} = \sqrt{x^2 - 10x + 25 + y^2}$

$d_{min} = \sqrt{x^2 - 10x + 25 + \left(\frac{x^2}{4} - 1\right)}$

$d_{min} = \left(\frac{5}{4}x^2 - 10x + 24\right)^{1/2}$

$d' = \frac{1}{2}\left(\frac{5}{4}x^2 - 10x + 24\right)^{-1/2}\left(\frac{5}{2}x - 10\right)$

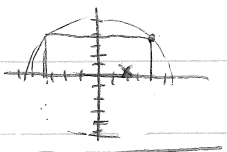
$0 = \frac{\frac{5}{2}x - 10}{2\left(\frac{5}{4}x^2 - 10x + 24\right)^{1/2}}$

$x = 4$

7. $y = \sqrt{25 - x^2}$

$y^2 = 25 - x^2$

$x^2 + y^2 = 25$



$y = \sqrt{25 - (3.536)^2}$

$y = 3.535$

dimensions of rect: 7.072×3.535

$A_{max} = lw$

$A_{max} = (2x)\left(\sqrt{25 - x^2}\right)$

$A_{max} = (2x)(25 - x^2)^{1/2}$

$A' = (2)(25-x^2)^{1/2} + (2x)\frac{1}{2}(25-x^2)^{-1/2}(-2x)$

$0 = 2(25-x^2)^{1/2} - 2x^2(25-x^2)^{-1/2}$

$0 = 2(25-x^2)^{-1/2}\left[(25-x^2) - x^2\right]$

$0 = \frac{2(25-2x^2)}{(25-x^2)^{1/2}}$

$x = 3.536$ $\quad x = \cancel{5, -5}$

8. $y = -x^2 + 2$ $(4, 0)$

$y = -(1.647)^2 + 2$

$y = -.713$

$(1.647, -.713)$

$d_{min} = \sqrt{(x-4)^2 + (y-0)^2}$

$d_{min} = \sqrt{x^2 - 8x + 16 + y^2}$

$d_{min} = \sqrt{x^2 - 8x + 16 + (-x^2+2)^2}$

$d_{min} = \sqrt{x^2 - 8x + 16 + x^4 - 4x^2 + 4}$

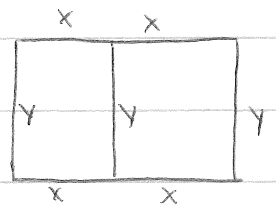
$d_{min} = (x^4 - 3x^2 - 8x + 20)^{1/2}$

$d' = \frac{1}{2}(x^4 - 3x^2 - 8x + 20)^{-1/2}(4x^3 - 6x - 8)$

$0 = \frac{4x^3 - 6x - 8}{2(x^4 - 3x^2 - 8x + 20)^{1/2}}$

$x = 1.647$

9.



$4x + 3y = 80$

$y = \frac{80 - 4x}{3}$

$y = \frac{80 - 4(10)}{3}$

$y = \frac{40}{3}$ ft.

$A_{max} = 2xy$

$A_{max} = 2x\left(\frac{80 - 4x}{3}\right)$

$A_{max} = \frac{160x - 8x^2}{3}$

$A_{max} = \frac{160}{3}x - \frac{8}{3}x^2$

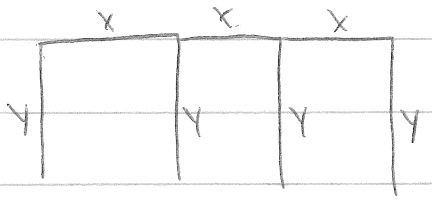
$A' = \frac{160}{3} - \frac{16}{3}x$

$\frac{16}{3}x = \frac{160}{3}$

$x = 10$ ft.

each: $10' \times \frac{40}{3}'$

10.



$2500 = 3xy$

$x = \frac{2500}{3y}$

$F_{min} = 3x + 4y$

$F_{min} = 3\left(\frac{2500}{3y}\right) + 4y$

$F_{min} = 2500y^{-1} + 4y$

$F' = -2500y^{-2} + 4$

$0 = \frac{-2500}{y^2} + 4$

$\frac{2500}{y^2} = 4$

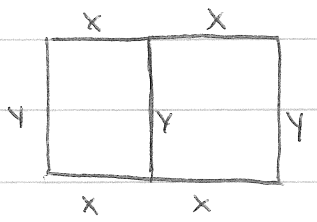
$y = \frac{50}{2} = 25$ ft.

$x = \frac{2500}{3(25)}$

$x = 33.333$ ft.

each $33\frac{1}{3}' \times 25'$

12.



$xy = 900$

$x = \frac{900}{y}$

$C_{min} = 2(4x) + 2(2y) + 1(4)$

$C_{min} = 8x + 5y$

$C_{min} = 8\left(\frac{900}{y}\right) + 5y$

$C_{min} = 7200y^{-1} + 5y$

$C' = -7200y^{-2} + 5$

$0 = \frac{-7200}{y^2} + 5$

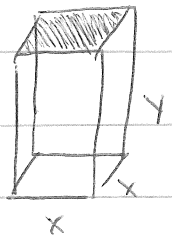
$\frac{7200}{y^2} = 5$

$y = 37.947$ ft.

$x = \frac{900}{(37.947)}$

$x = 23.717$ ft.

pen: 47.434 ft. $\times$ 37.947 ft.

13. 

$x^2 + 4(xy) = 1000$

$4xy = 1000 - x^2$

$y = \frac{1000 - x^2}{4x}$

$y = \frac{1000 - (18.257)^2}{4(18.257)}$

$y = 9.129$ ft.

$V_{max} = x^2 y$

$V_{max} = x^2 \left(\frac{1000 - x^2}{4x}\right)$

$V_{max} = \frac{1000x^2 - x^4}{4x}$

$V_{max} = 250x - \frac{1}{4}x^3$

$V' = 250 - \frac{3}{4}x^2$

$0 = 250 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 250$

$x^2 = \frac{1000}{3}$

$x = 18.257$ ft.

$18.257' \times 18.257' \times 9.129'$